THE MELODY OF MATH

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Background Research:

First, some background knowledge about sound waves and music must be established. Musical sound is composed of sound waves, and therefore is measured in Hertz (Hz). A higher pitched sound will correspond to a higher frequency of wave and therefore a higher measure of hertz.

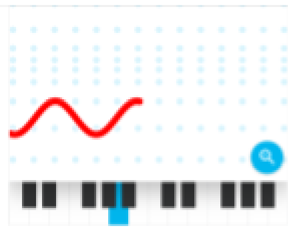

Figure 1

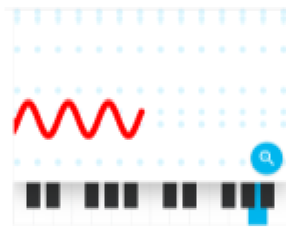

Figure 1

Each musical note on a staff has a set frequency. Most of the modern world operates at the tuning standard A4=440 Hz, but now many modern orchestras are tuning to a frequency as high as A4=444. The pitch standard was instituted by The International Organization for Standardization in 1955, but has been debated since. There are some arguments for tuning to A4=432, with claims saying that it produces a clearer and brighter tone while helping to ground listeners, but no evidence has been produced to back this up. Similarly, some solo violinists choose to tune to A4=442, just 2 Hz more than the standard, to help them stand out from the orchestra and produce a brighter sound.

The number 4 directly next to the A signifies which octave it is in on a musical staff (higher vs. lower). An A5 (exactly one octave higher) will be double the previous frequency, at 880 Hz.

Creating Sound

Sound is transmitted through the air, essentially making it an alternation of high and low pressure air. *Figure 3a* shows the compression of air molecules from the source of sound through the air. Rather than complete movement of a singular air molecule from one side to the other, a rippling motion is created, as each one pushes into another. *Figure 3b* depicts the way these

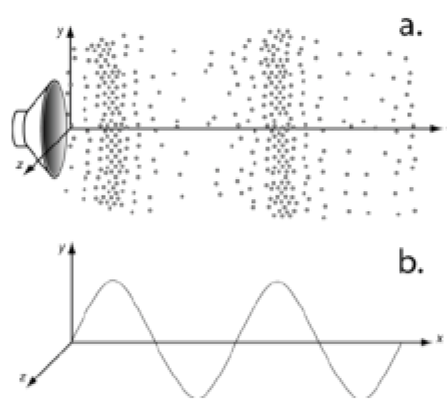

Figure 3

longitudinal waves are translated into transverse waves. In other words, actual air molecules resemble an accordion when being compressed and pulled, while sound waves are shown with movement similar to ocean waves. When put into a graph, the x-axis will represent time, while the y-axis will represent pressure/density. The more dense areas of the air molecules will be shown as a crest on the wave (high point), while the low pressure areas are depicted with a trough (low point). If *Figure 3b* was not a wave, and instead a straight line, this would indicate no sound because there is no change in air pressure.

The Human Ear

The ear can be divided into 3 categories as seen in *Figure 4*: outer, middle, and inner ear (cochlea). Sound waves move through the external auditory canal and vibrate in the tympanic membrane (ear drum). This vibration moves three small bones (ossicles) meant to amplify the waves and pass them to the cochlea. There, the waves of air molecules are finally changed to waves of endolymph (fluid in the cochlea). Different sections of the cochlea respond to specific frequencies so it can work with the brain to interpret sound.

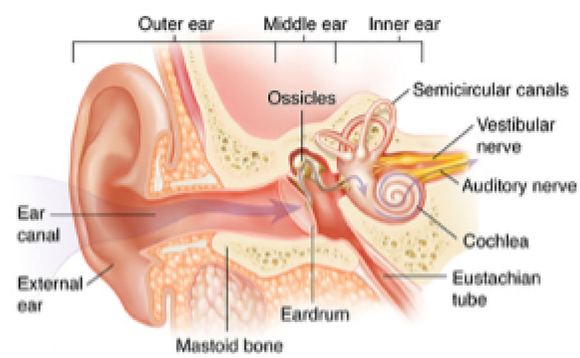

Figure 4

Instrument Timbre

Timbre can be defined as the quality of sound produced from an instrument. Size, material, and shape of an instrument all have different effects on sound wave quality- shown in *Figure 5.* Sound is made from vibration: a bow on a violin or circulating air on a wind instrument (ex. flute).

Some instruments have an open cavity to amplify the vibration, such as a guitar or a ukulele. The tone of the instrument will have an effect on the overall shape of the wave, as different patterns can be observed based on the instrument.

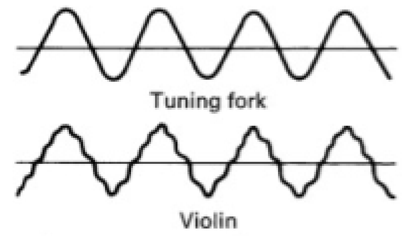

Figure 5

A tuning fork is designed to play a pure tone so that other instruments can match it. Because of this, the sound wave will be smooth. However, a violin is made of wood, and strings are utilized to make vibration, so the wave is less pure and has more ridges, giving it a unique sound. A flute is made of metal, making the sound wave very different from a violin. In addition, flutists use a technique called vibrato (from the Italian word "to vibrate"). In this technique, players will fluctuate the tuning of one note slightly above and below the actual pitch, quickly for high notes and slower for lower notes. Vibrato can be described as a pulsating or wobbling tone, adding emotion to the music played. Utilization of this can be seen at the crest of each flute sound wave in *Figure 5,* a slight vibration smaller than the actual sound wave.

Last, overtones, which will be discussed later, impact the quality or feel of the produced sound. A violin produces frequencies with many overtones, creating a rich sound. In contrast, a flute creates less overtones, contributing to the clear tone it is characterized by. Interestingly, the "woody" tone of a clarinet comes from the amount of lower overtones.

When instruments play the same notes, the sound waves will always look different. Even though the general outlines can match up, the small ridges and "imperfections" make the instruments unique.

Changing Pitch

Pitch is changed by making the wavelength shorter or longer. It can look like pressing down strings on a violin (making them shorter) which corresponds to a shorter wavelength, higher frequency, and higher pitch. In addition, covering holes on a flute makes the distance the air travels longer, which creates a lower sound. All instruments follow the logic of changing the vibration through physical manipulation, including trumpet. For example, pressing down one valve on a trumpet lowers the pitch one full step on a musical staff. The phenomenon can be seen even in a simple rubber band. Stretching it tighter will make a higher sound when plucked because the vibration will be faster.

Consonance and Dissonance

"Consonant" is an adjective used to describe the relationship between two notes, and specifically is used when they complement each other. Dissonance, on the other hand, is the opposite, created when two notes clash or sound harsh when played at the same time. Different intervals will cause either one of these effects. An interval is the difference between two notes. A "third" is 3 full steps apart (shown in Figure 6).

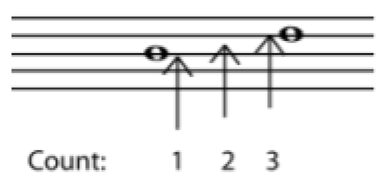

Figure 6

Consonance is the structure of most songs. Using likable sounds will increase the chance of listeners. It creates a pleasant and stable song. Even so, songs can become boring with minimal change of chords. Because of this, dissonance is needed. It will create tension in a song. Clashing sounds will draw listeners in, and keep them engaged if these suspenseful chords are eventually resolved with consonance. Consonance and dissonance rely on each other to create a pleasing and complete song.

Consonance and dissonance are on every composer's mind when writing music. They know how to manipulate intervals to create the needed effect. For example the soundtrack of the movie "Jaws" utilizes dissonance to create an air of darkness and anticipation. To give a classical example, Mozart's String Quartet No. 19 is oftentimes nicknamed "Dissonant", and uses it to emphasize lyricism and sadness. However, the song does eventually resolve with consonance to relieve the tension and create a sense of resolution or closure.

Creating Consonance

The idea of consonance is widely agreed upon in the western hemisphere, but may vary in other places based on culture. In most cases, it includes the minor/major third, the perfect fourth, the perfect fifth, and octave (unison). These describe how far apart notes are located on a staff. These notes interact well with each other due to their overtones. An overtone is any tone that sounds above the fundamental tone (the frequency assigned to that note). Using the previous example, an A4 (with a frequency of 440Hz) would have overtones of 880 Hz, 1320 Hz, 1760 Hz, and so on. These values are derived by multiplying the fundamental frequency with whole numbers. The fundamental tone will always be heard, but with practice, the listener is able to hear overtones. Consonance appears when notes are harmonically related to each other (i.e. when one note is another's overtone). However, because sound waves are depicted as sine functions, consonance can

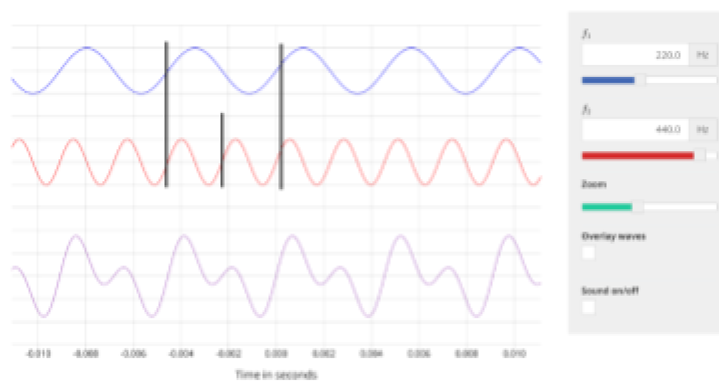

Figure 7a

be seen as occurring simply when the periods of the two waves line up more frequently (shown in Figure 7a).

Creating Dissonance

A "complicated" wavelength ratio creates dissonance. For example, a 2:1 ratio will create an octave interval, a simple ratio and a consonant tone. But, ratios such as 15:16 (minor second) or 32:45 (tritone) clash more. These ratios describe the fact that the waves will have to oscillate more times in order to line up again.

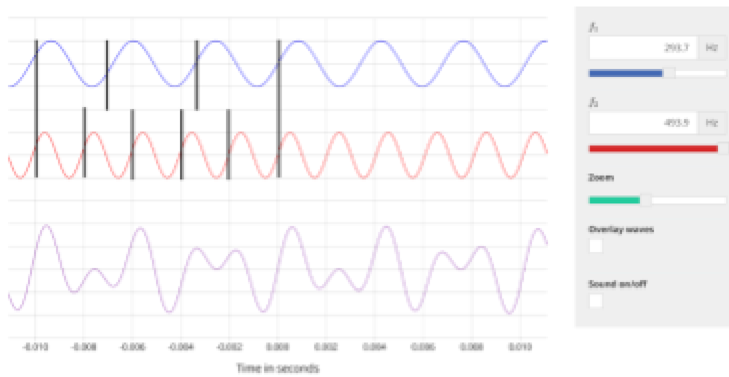

Figure 7b

Think of two ticking clocks: one that ticks 32 times per minute and another that ticks 45 times per minute. If both clocks start at the same time, they are going to take a long time to sync up. On the other hand, if one clock ticks 2 times per minute and the other ticks once per minute (representing the 2:1 ratio of an octave interval) there is only one instance where the clocks are ticking at different times, making the interval more consonant. Other examples include the major second, tritone, minor seventh, and major seventh. This is seen in Figure 7b, as the top wave (293.7 Hz) completes 4 periods before it lines up with the second wave (493.9 HZ) after 5 full periods. The repeat time (0.01 seconds) is very long compared to the octave interval depicted in figure 7a (0.005 seconds). Because the time it takes for the wave to line up again is double the time of the octave interval, the interval produced sounds much more dissonant.

Final Thoughts

The unique sound of music can be attributed to many interrelated factors that come together to create a well-crafted song. First and foremost, the variations in the way an orchestra is tuned will alter the feel of a song. Second, the material of an instrument, the way it is constructed, and the technique used to produce the sound are all very impactful upon instrument timbre, and these nuances shape the music into something remarkable. Last, the intentional use and fluctuation between consonance and dissonance that relies on mathematical concepts such as ratios and intervals is what makes a song complete. The connection between math and music doesn't just explain confusing concepts, but enriches appreciation of such a complicated form of art.

Further Directions

There is still much to learn about how the interaction of sound waves can explain the way music sounds. A topic of further research could be investigating pitched vs. unpitched sounds and instruments and how they can create contrast within a song. Focusing on specifically percussion, instruments like marimbas or xylophones are pitched and create sound waves, while a snare drum doesn't have any distinct pitches. Another interesting topic is how sound waves travel through different materials. For instance, sound waves travel fastest through solids, then liquids, and lastly through gases. This is because the molecules are compacted the tightest in solids, so when they vibrate, the vibration passes through very quickly.

Works Cited

"Anatomy of the Ear." *Saint Luke's Health System*, www.saintlukeskc.org/health-library/anatomy-ear.

Andrews, David R. "Middle c - an Overview | ScienceDirect Topics." *Www.sciencedirect.com*, 2003, www.sciencedirect.com/topics/mathematics/middle-c.

Bennett II, James. "Why Do Orchestras Tune to an A-Note Pitch at 440 Hz? | How to Classical." *WQXR*, 5 July 2017, www.wqxr.org/story/why-do-we-tune-a-note-pitch/.

"EarMaster - Music Theory & Ear Training on PC, Mac, IPad and IPhone." *Www.earmaster.com*, www.earmaster.com/music-theory-online/ch04/chapter-4-5.html.

"Overtone | Acoustics." *Encyclopedia Britannica*, 3 Oct. 2024, www.britannica.com/science/overtone.

Podolefsky , Noah S. , and Noah D. Finkelstein. "Figure 1. A Sound Wave Represented by a Picture of Compressed And..." *ResearchGate*, 2017, www.researchgate.net/figure/A-sound-wave-represented-by-a-picture-of-compressed-and-rarefied-air-particles-a-and-a_fig1_26493787. Accessed 13 Oct. 2024.

Sheldon, Robert. "What Is Hertz? - Definition from WhatIs.com." *SearchMobileComputing*, www.techtarget.com/searchmobilecomputing/definition/hertz.

"Transverse and Longitudinal Waves." *Web.fscj.edu*, web.fscj.edu/Milczanowski/psc/lect/Ch6/slide2.htm.